Class XII

Mathematics

Test-1

MAX –MARKS-100   TIME: 3.00 Hr

1.   If A is a square matrix of order 3 such that $|adj\,A| = 64$, find $|A|$.

2.   lf A, B, C are three nonzero square matrices of same order, find the condition on A such that $AB=AC \Rightarrow B=C$.

3.   Give an example of two non zero 2×2 matrices A, B such that AB = 0.

4.   lf $f(1) = 4$; $f'(1) = 2$, find the value of the derivative of $\log f(e^x)$ w.r.t. x at the point $x = 0$.

5.   Find a, for which $f(x) = a(x + \sin x) + a$ is increasing.

6.   Evaluate, $\int_0^{1.5} [x] dx$ (where [x] is greatest integer function)

7.   Write the order and degree of the differential equation,
$$y = \frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

8.   If $\vec{a} = \hat{i} + \hat{j}$; $\vec{b} = \hat{j} + \hat{k}$; $\vec{c} = \hat{k} + \hat{i}$, find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$

9.   A four digit number is formed using the digits 1,2,3,5 with no repetitions. Find the probability that the number is divisible by 5.

10.   The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of the experiment are performed. Find the probability that the event happens at least once.

SECTION - B

11. Find the value of $2\tan^{-1}\left(\dfrac{1}{5}\right) + \sec^{-1}\left(\dfrac{5\sqrt{2}}{7}\right) + 2\tan^{-1}\dfrac{1}{8}$

12. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI + bA)^n = a^n.I + na^{n-1}bA$ where I is a unit matrix of order 2 and n is a positive integer

OR

Using properties of determinants, prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

13. If $x = a\sin pt$ and $y = b\cos pt$, find the value of $\dfrac{d^2y}{dx^2}$ at $t = 0$.

Find the equations of tangent lines to the curve $y = 4x^3 - 3x + 5$ which are perpendicular to the line $9y + x + 3 = 0$.

14. Show that the function $f(x) = |x+2|$ is continuous at every $x \in \mathbf{R}$ but fails to be differentiable at $x = -2$.

15. Evaluate $\displaystyle\int \dfrac{x^2 + 4}{x^4 + x^2 + 16}\,dx$

16. Evaluate $\displaystyle\int_0^{\pi/2} \dfrac{x + \sin x}{1 + \cos x}\,dx$ OR $\displaystyle\int \dfrac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}\,dx$.

17. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A,B,C of □ABC respectively. Find an expression for the area of □ABC and hence deduce the condition for the points A,B,C to be collinear.

OR

Show that the points A,B,C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$

and $3\hat{i} - 4\hat{j} - 4\hat{k}$ are the vertices of a right triangle. Also find the remaining angles of the triangle.

18. Evaluate $\int_0^\pi \dfrac{x}{\sqrt{a^2 \cos^2 x + b^2 \sin^2 x}} dx$ $a, b > 0$.

19. Solve the differential equation, $\dfrac{dy}{dx} + y \sec^2 x = \tan x \sec^2 x$; $y(0) = 1$

OR

$2xy + y^2 - 2x^2 \dfrac{dy}{dx} = 0$; $y(1) = 2$

20. In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product.

   (i) What is the probability that the bolt drawn is defective?

   (ii) If the bolt is found to be defective find the probability that it is a product of machine B.

21. Two dice are thrown simultaneously. Let X denote the number of sixes, find the probability distribution Also find the mean and variance of X, using the probability distribution table.

SECTION-C

22. Consider $f : R \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible. Find the inverse of f.

23. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.**OR**

The cost of fuel for running a bus is proportional to the square of the speed generated in km/h. It costs Rs.48/hr, When the bus is moving with

a speed of 20km/h. what is the most economical speed if the fixed charges are Rs.108 for one hour, over and above the running charges.

24. Make a rough sketch of the region given below and find it's area using integration $\{(x,y); y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$.

25. Every grain of wheat provides 0.1 gin of proteins and 0.25gm of carbohydrates. The corresponding values for rice are 0.05gm and 0.5gm respectively. Wheat costs Rs.4 per kg and rice Rs.6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 50 gins and 200 gins respectively. ln what quantifies should wheat and rice be mixed in the daily diet to provide minimum daily requirements of proteins and carbohydrates at minimum cost. Frame an L.P.P and solve it graphically.

26. font $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$$x + 2y + z = 4$$
$$-x + y + z = 0$$
$$x - 3y + z = 2$$

27. Find the equation of the plane containing the lines,

$$\vec{r} = i + j + \lambda(\hat{i} + 2\hat{j} - \hat{k}) \text{ and } \vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$$

Find the image of (1,2,3) on this plane .

OR

Find the equation of the plane passing through the intersection of the planes, $2x + 3y - z + 1 = 0$; $x + y - 2z + 3 = 0$ and perpendicular the plane $3x - y - 2z - 4 = 0$. Also find the inclination of this plane with the xy plane.

28. Prove that $\int\limits_{0}^{\frac{\pi}{2}} \log(\sin x)dx = \int\limits_{0}^{\frac{\pi}{2}} \log(\cos x)dx = \frac{-\pi}{2}\log 2$

Assignment by: AJAY KAKKAR 9811400749----9899333500--- 01132929394